International Journal of Bio-resource and Stress Management

https://pphouse.org/ijbsm.php

Crossref

IJBSM March 2022, 13(3):276-282 Print ISSN 0976-3988 Online ISSN 0976-4038 Article AR2771a

Review Article Natural Resource Management

DOI: HTTPS://DOI.ORG/10.23910/1.2022.2771a

Some Efficient Methods to Remove Bias in Ratio and Product Types Estimators in Ranked Set Sampling

Nitu Mehta[1] **and V. L. Mandowara**[2]

[1]Dept. of Agricultural Economics & Management, Rajasthan College of Agriculture, Maharana Pratap University of Agriculture and Technology, Udaipur, Rajasthan (313 001), India

[2]Dept. of Mathematics & Statistics, University College of Science, M. L. Sukhadia University, Udaipur, Rajasthan (313 001), India

Open Access

Corresponding nitumehta82@gmail.com

0000-0002-0883-3467

ABSTRACT

Ranked set sampling is one method to potentially increase precision and reduce costs by using quantitative or qualitative information to obtain a more representative sample. Use of auxiliary information has shown its significance in improvement of efficiency of estimators of unknown population parameters. Ratio estimator is used when auxiliary information in the form of population mean of auxiliary variable at estimation stage for the estimation of population parameters when study and auxiliary variable are positively correlated. In case of negative correlation between study variable and auxiliary variable, Product estimator is defined for the estimation of population mean. This paper proposed the problem of reducing the bias of the ratio and product estimators of the population mean in ranked set sampling (RSS). This paper suggested several type unbiased estimators of the finite population mean using information on known population parameters of the auxiliary variable in ranked set sampling. An important objective in any statistical estimation procedure is to obtain the estimators of parameters of interest with more precision. The Variance of the proposed unbiased ratio and product estimators are obtained up to first degree of approximation. Theoretically, it is shown that these suggested estimators are more efficient than the unbiased estimators in Simple random sampling. A numerical illustration is also carried out to demonstrate the merits of the proposed estimators using RSS over the usual estimators in SRS.

KEYWORDS: RSS, unbiased estimators, ratio product estimators, auxiliary variables

***Citation* (VANCOUVER):** Mehta and Mandowara, Some Efficient Methods to remove Bias in Ratio and Product types Estimators in Ranked Set Sampling. *International Journal of Bio-resource and Stress Management,* 2022; 13(3), 276-282. HTTPS://DOI.ORG/10.23910/1.2022.2771a.

Copyright: © 2022 Mehta and Mandowara. This is an open access article that permits unrestricted use, distribution and reproduction in any medium after the author(s) and source are credited.

Data Availability Statement: Legal restrictions are imposed on the public sharing of raw data. However, authors have full right to transfer or share the data in raw form upon request subject to either meeting the conditions of the original consents and the original research study. Further, access of data needs to meet whether the user complies with the ethical and legal obligations as data controllers to allow for secondary use of the data outside of the original study.

Conflict of interests: The authors have declared that no conflict of interest exists.

RECEIVED on 07th December 2021 RECEIVED in revised form on 29th February 2022 ACCEPTED in final form on 25th March 2022 PUBLISHED on 31st March 2022

1. INTRODUCTION

The literature on Ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling was first suggested by McIntyre (1952) to increase the efficiency of estimator of population mean. Kadilar et al. (2009) used this technique to improve ratio estimator given by Prasad (1989) and Bouza (2008) used this to improve the product estimator. Mandowara and Mehta (2013) suggested efficient generalized Ratio-Product type estimators using RSS. Jeelani and Bouza (2015) suggested new ratio method of estimation under ranked set sampling. Mehta and Mandowara (2012, 2016) proposed a better estimator of population mean with power transformation and modified ratio-cum-product estimator in RSS. Here we shall propose two modified Methods to construct unbiased ratio and product type estimators of population mean using Ranked set sampling based on auxiliary variable. The Variance of the proposed unbiased ratio and product tpye estimators are obtained up to first degree of approximation.

The classical ratio and product estimators given by Cochran (1940) and Murthy (1964) respectively for estimating the population mean $\bar{Y}$, respectively for SRS, are defined as

$$\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \quad (1.1)$$

$$\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) \quad (1.2)$$

We have observed that the ratio and product type estimators are biased. Several researchers have an attempted to reduce the bias from these estimators. Quenouille (1956) considered an estimator of the population mean $\bar{Y}$ as

$$\bar{y}_Q = a(\bar{y}_{R_1} + \bar{y}_{R_2}) + (1-2a)\bar{y}_R \quad (1.3)$$

Where a is a suitably chosen constant such that the bias in the estimator $\bar{y}_Q$ is zero. Here

a) $\bar{y}_{R_1} = \bar{y}_1\left(\frac{\bar{X}}{\bar{x}_1}\right)$, where $\bar{y}_1 = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the first half sample means for Y and X variables respectively.

b) $\bar{y}_{R_2} = \bar{y}_2\left(\frac{\bar{X}}{\bar{x}_2}\right)$, where $\bar{y}_2 = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x}_2 = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the second half sample means for Y and X variables respectively.

c) $\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$, where $\bar{y} = \frac{1}{2n}\sum_{i=1}^{2n} y_i$ and $\bar{x} = \frac{1}{2n}\sum_{i=1}^{2n} x_i$ are the sample means for Y and X variables respectively, based on the pooled samples.

This Quenouille estimator $\bar{y}_Q$ is an unbiased estimator of population mean $\bar{Y}$ if. $a = -\frac{(N-n)}{2N}$

Hartley and Ross (1954) suggested ingenious methods of proposing an exactly unbiased ratio type estimator $\bar{y}_{HR}$ and variance of the estimator of population mean $\bar{Y}$ is given by

$$\bar{y}_{HR} = \bar{r}\bar{X} + \frac{(N-1)}{N}\frac{n}{(n-1)}\left(\bar{y} - \bar{r}\bar{x}\right) \quad (1.4)$$

Here $\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{x_i}$

To the first degree of approximation, variance of the estimator $\bar{y}_{HR}$ is given by

$$V(\bar{y}_{HR}) = \theta[S_y^2 + \bar{R}^2 S_x^2 - 2\bar{R}S_{xy}], \text{ where } \theta = \frac{1}{n} (\text{ignoring } f = \frac{n}{N}) \quad (1.5)$$

Motivated by Hartley and Ross (1954), Sarjinder singh (2003) suggested the unbiased product type estimator of the population mean $\bar{Y}$ is given by

$$\bar{y}_{pu} = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right) - \left(\frac{1-f}{n}\right)\frac{s_{xy}}{\bar{X}} \quad (1.6)$$

The variance $V(\bar{y}_{pu})$ to the first order of approximation is given as

$$V(\bar{y}_{pu}) = \theta[S_y^2 + \bar{R}^2 S_x^2 + 2\bar{R}S_{xy}] \quad (1.7)$$

In Ranked set sampling (RSS), m independent random sets are chosen, each of size m and units in each set are selected with equal probability and without replacement from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the m^{th} set. This cycle may be repeated r times, so mr $(=n)$ units have been measured during this process.

When we rank on the auxiliary variable, let $(y_{[i]}, x_{(i)})$ denote the i^{th} judgment ordering for the study variable and i^{th} perfect ordering for the auxiliary variable in the i^{th} set, where $i=1,2,3.........., m$.

Swami (1996) and Bouza (2008) defined the ratio and product estimators for the population mean in ranked set sampling as

$$\bar{y}_{R,RSS} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right), \quad (1.8)$$

$$\bar{y}_{P,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right), \quad (1.9)$$

Where $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$, $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the ranked set sample means for variables y and x respectively.

To the first degree of approximation the bias and mean squared error (MSE) of the estimator $\bar{y}_{R,RSS}$ and $\bar{y}_{P,RSS}$ are given as

$$B(\bar{y}_{R,RSS}) = \bar{Y}\left[\theta(C_x^2 - \rho_{yx}C_yC_x) - (W_{x(i)}^2 - W_{yx(i)})\right] \quad (1.10)$$

$$B(\bar{y}_{P,RSS}) = \bar{Y}\left\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\right\} \quad (1.11)$$

$$MSE(\bar{y}_{R,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\} - \{W_{y[i]} - W_{x(i)}\}^2\right], \quad (1.12)$$

$$MSE(\bar{y}_{P,RSS}) = \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\} - \{W_{y[i]} + W_{x(i)}\}^2\right]. \quad (1.13)$$

where $\theta = \frac{1}{mr}$, $\varepsilon_0 = \frac{\bar{y}_{[n]} - \bar{Y}}{\bar{Y}}$, $\varepsilon_1 = \frac{\bar{x}_{(n)} - \bar{X}}{\bar{X}}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$, $C_{yx} = \frac{S_{yx}}{\bar{X}\bar{Y}} = \rho_{yx}C_yC_x$, $W_{x(i)}^2 = \frac{1}{m^2r}\frac{1}{\bar{X}^2}\sum_{i=1}^{m}\tau_{x(i)}^2$, $W_{y[i]}^2 = \frac{1}{m^2r}\frac{1}{\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2$, and $W_{yx(i)} = \frac{1}{m^2r}\frac{1}{\bar{X}\bar{Y}}\sum_{i=1}^{m}\tau_{yx(i)}$. Here we would like to remind $\tau_{x(i)} = \mu_{x(i)} - \bar{X}$, $\tau_{y[i]} = \mu_{y[i]} - \bar{Y}$ and $\tau_{yx(i)} = (\mu_{x(i)} - \bar{X})(\mu_{y[i]} - \bar{Y})$.

2. QUENOUILLE'S METHOD IN RANKED SET SAMPLING

In this section, we apply the technique of Quenouille (1956) to Ranked set sampling to reduce bias from ratio estimator and constructing the unbiased ratio estimator of the population mean. Here we draw a sample of size 2n units from a population of N units. We divide the sample of 2n units into two equal halves each of size n. The sample based on 2n units is called the pooled sample. Then we have three biased ratio estimators of the population mean as

a) $\bar{y}_{R_{1,RSS}} = \bar{y}_{[n_1]}\left(\frac{\bar{X}}{\bar{x}_{(n_1)}}\right)$, where $\bar{y}_{[n_1]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n_1)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the first half sample means for Y and X variables respectively.

b) $\bar{y}_{R_{2,RSS}} = \bar{y}_{[n_2]}\left(\frac{\bar{X}}{\bar{x}_{(n_2)}}\right)$, where $\bar{y}_{[n_2]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n_2)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$ are the second half sample means for Y and X variables respectively.

c) $\bar{y}_{R_{RSS}} = \bar{y}_{[n]}\left(\frac{\bar{X}}{\bar{x}_{(n)}}\right)$, where $\bar{y}_{[n]} = \frac{1}{2n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n)} = \frac{1}{2n}\sum_{i=1}^{n} x_{(i)}$ are the sample means for Y and X variables respectively, based on the pooled samples.

By following the ratio method of the population mean in ranked set sampling, we have

$$E(\bar{y}_{R_{1,RSS}}) = \bar{Y} + \bar{Y}\left[\theta(C_x^2 - \rho_{yx}C_yC_x) - \{W_{x(i)}^2 - W_{yx(i)}\}\right] \quad (2.1)$$

$$E(\bar{y}_{R_{2,RSS}}) = \bar{Y} + \bar{Y}\left[\theta(C_x^2 - \rho_{yx}C_yC_x) - \{W_{x(i)}^2 - W_{yx(i)}\}\right] \quad (2.2)$$

$$E(\bar{y}_{R_{RSS}}) = \bar{Y} + \bar{Y}\left[\frac{\theta}{2}(C_x^2 - \rho_{yx}C_yC_x) - \{W_{x(i)}^2 - W_{yx(i)}\}\right] \quad (2.3)$$

Quenoulli's estimator of the population mean $\bar{Y}$ under RSS as,

$$\bar{y}_{Q,RSS} = a(\bar{y}_{R_{1,RSS}} + \bar{y}_{R_{2,RSS}}) + (1-2a)\bar{y}_{R_{RSS}} \quad (2.4)$$

Where a is a suitably chosen constant such that the bias in the estimator is zero.

$$E(\bar{y}_{Q,RSS}) = E[a(\bar{y}_{R_{1,RSS}} + \bar{y}_{R_{2,RSS}}) + (1-2a)\bar{y}_{R_{RSS}}]$$

$$E(\bar{y}_{Q,RSS}) = \bar{Y} + \bar{Y}\left[\{(C_x^2 - \rho_{yx}C_yC_x) - (W_{x(i)}^2 - W_{yx(i)})\}\left\{2a\frac{1}{\theta} + (1-2a)\frac{1}{2\theta}\right\}\right]$$

Evidently the bias in the estimator $\bar{y}_{Q,RSS}$ will be zero if or if

$$2a\frac{1}{\theta} + (1-2a)\frac{1}{2\theta} = 0$$

$$\Rightarrow \frac{a}{\theta} + \frac{1}{2\theta} = 0 \text{ or if } a = -\frac{1}{2}$$

3. PROPOSED EXACTLY UNBIASED RATIO TYPE ESTIMATOR IN RSS

Motivated by Hartley and Ross (1954), we suggest a method of proposing an exactly unbiased ratio-type estimator for the population mean $\bar{Y}$ using RSS. In RSS, let us define

the ratio $r_{(i)} = \frac{y_{[i]}}{x_{(i)}}$ and sample mean of the ratio as

$$\bar{r}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} r_{(i)} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_{[i]}}{x_{(i)}}$$

We will define a ratio estimator of the population mean as

$$\bar{y}_{R,RSS} = \bar{r}_{(n)}\bar{X}$$

Taking expected values on both sides, we obtain

$$E(\bar{y}_{R,RSS}) = E(\bar{r}_{(n)}\bar{X}) = \bar{X}\frac{1}{N}\sum_{i=1}^{N}\frac{Y_i}{X_i} \neq \bar{Y}$$

Note that $\bar{R} = \frac{1}{N}\sum_{i=1}^{N}\frac{Y_i}{X_i}$ and $R_i = \frac{Y_i}{X_i}$ therefore the bias in the estimator $\bar{y}_{R,RSS}$ is

$$B(\bar{y}_{R,RSS}) = E(\bar{y}_{R,RSS}) - \bar{Y}$$

$$= \bar{X}\bar{R} - \frac{1}{N}\sum_{i=1}^{N} R_iX_i = -\frac{(N-1)}{N}S_{rx}$$

Where $S_{rx} = \frac{1}{(N-1)}\left[\sum_{i=1}^{N} R_iX_i - N\bar{R}\bar{X}\right] = \frac{N}{N-1}\left[\bar{Y} - \bar{R}\bar{X}\right]$

Thus an estimator of S_{rx} is given by

$$s_{rx} = \frac{n}{(n-1)}\left[\bar{y}_{[n]} - \bar{r}_{[n]}\bar{x}_{(n)}\right]$$

Hence an estimator of $B(\bar{y}_{R,RSS})$ is

$$\hat{B}(\bar{y}_{R,RSS}) = -\left(\frac{N-1}{N}\right)\frac{n}{(n-1)}\left[\bar{y}_{[n]} - \bar{r}_{(n)}\bar{x}_{(n)}\right]$$

Here, we suggest an unbiased ratio type estimator of the population mean $\bar{Y}$ in RSS is

$$\bar{y}_{MM1,RSS} = \bar{r}_{[n]}\bar{X} + \frac{(N-1)}{N}\frac{n}{(n-1)}\left(\bar{y}_{[n]} - \bar{r}_{(n)}\bar{x}_{(n)}\right) \quad (3.1)$$

Taking expectation of both sides, we get

$$E(\bar{y}_{MM1,RSS}) = \bar{X}E\left(\frac{1}{n}\sum_{i=1}^{n}\frac{y_{[i]}}{x_{(i)}}\right) + \frac{(N-1)}{N}E(s_{rx})$$

$$= \bar{X}\left(\frac{1}{N}\sum_{i=1}^{N}\frac{Y_i}{X_i}\right) + \frac{(N-1)}{N}S_{rs}$$

Putting the value of S_{rx}, we obtained

$$E(\bar{y}_{MM1,RSS}) = \bar{Y} \quad (3.2)$$

For large values of n and N, the estimator $\bar{y}_{MM1,RSS}$ can be

$$\bar{y}_{MM1,RSS} \approx \bar{r}_{[n]}\bar{X} + \left(\bar{y}_{[n]} - \bar{r}_{(n)}\bar{x}_{(n)}\right) = \bar{y}_{[n]} + \bar{r}_{(n)}\left(\bar{X} - \bar{x}_{(n)}\right)$$

Defining $\tau = \frac{\bar{r}_{(n)}}{\bar{R}} - 1$ such that $E(\tau) = 0$

$$\Rightarrow \bar{r}_{(n)} = \bar{R}(\tau + 1)$$

To obtain variance of $\bar{y}_{MM1,RSS}$, we put $\bar{y}_{[n]} = \bar{Y}(1+\varepsilon_0)$ and $\bar{x}_{(n)} = \bar{X}(1+\varepsilon_1)$ so that $E(\varepsilon_0) = E(\varepsilon_1) = 0$, and therefore,

$$V(\varepsilon_0) = E(\varepsilon_0^2) = \frac{V(\bar{y}_{[n]})}{\bar{Y}^2}$$

$$= \frac{1}{mr}\frac{1}{\bar{Y}^2}\left[S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right] = \left[\theta C_y^2 - W_{y[i]}^2\right]$$

Similarly, $V(\varepsilon_1) = E(\varepsilon_1^2) = \left[\theta C_x^2 - W_{x(i)}^2\right]$

$$Cov(\varepsilon_0, \varepsilon_1) = E(\varepsilon_0, \varepsilon_1) = \frac{Cov\left(\bar{y}_{[n]}, \bar{x}_{(n)}\right)}{\bar{X}\bar{Y}}$$

$$= \frac{1}{\bar{X}\bar{Y}}\frac{1}{mr}\left[S_{yx} - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right] = \left[\theta\rho_{yx}C_yC_x - W_{yx(i)}\right]$$

The estimator $\bar{y}_{MM1,RSS}$ in terms of ε_0, ε_1 and τ can be written as

$$\bar{y}_{MM1,RSS} = \bar{Y}(1+\varepsilon_0) - \bar{R}\bar{X}\varepsilon_1(1+\tau) \qquad (3.3)$$

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\varepsilon_0|$ and $|\varepsilon_1|$ as small so that the terms involving ε_0 and or ε_1 in a degree greater than two will be negligible.

Now, the variance of an unbiased estimator $\bar{y}_{MM1,RSS}$ is given ,

$$V(\bar{y}_{MM1,RSS}) = E[\bar{y}_{MM1,RSS} - E(\bar{y}_{MM1,RSS})]^2$$

$$\approx E[\bar{Y}(1+\varepsilon_0) - \bar{R}\bar{X}\varepsilon_1 - \bar{Y}]^2$$

$$= E[\bar{Y}\varepsilon_0 - \bar{R}\bar{X}\varepsilon_1]^2$$

$$= [\bar{Y}^2\{\theta C_y^2 - W_{y[i]}^2\} + \bar{R}^2\bar{X}^2\{\theta C_x^2 - W_{x(i)}^2\} - 2\bar{R}\bar{X}\bar{Y}\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\}]$$

$$\Rightarrow V(\bar{y}_{MM1,RSS}) = \theta\{S_y^2 + \bar{R}^2S_x^2 - 2\bar{R}S_{yx}\} - \{\bar{Y}W_{y[i]} - \bar{R}\bar{X}W_{x(i)}\}^2 \quad (3.4)$$

4. SUGGESTED EXACTLY UNBIASED PRODUCT TYPE ESTIMATOR IN RSS

Adapting the estimator given by Singh Sarjinder (2003) and utilizing the product estimator for the population mean in ranked set sampling given by Bouza (2008), we suggest a method of proposing an exactly unbiased Product type estimator for the population mean $\bar{Y}$ using RSS.

Bouza (2008) defined the product estimator for the population mean in ranked set sampling as

$$\bar{y}_{P,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right)$$

To the first degree of approximation the bias of the estimator $\bar{y}_{P,RSS}$ is given as

$$B(\bar{y}_{P,RSS}) = \bar{Y}\{\theta\rho_{yx}C_yC_x - W_{yx(i)}\}$$

$$= \frac{1}{mr}\frac{S_{xy}}{\bar{X}} - \bar{Y}W_{yx(i)}$$

Thus an unbiased estimator of bias in the product estimator is given by

$$\hat{B}(\bar{y}_{P,RSS}) = \frac{1}{mr}\frac{s_{xy}}{\bar{X}} - \bar{Y}W_{yx(i)}$$

Here, we suggest an unbiased product type estimator of the population mean $\bar{Y}$ in RSS is given as

$$\bar{y}_{MM2,RSS} = \bar{y}_{[n]}\left(\frac{\bar{x}_{(n)}}{\bar{X}}\right) - \frac{1}{mr}\frac{s_{xy}}{\bar{X}} + \bar{Y}W_{yx(i)} \qquad (4.1)$$

Defining $\varepsilon_4 = \frac{s_{xy}}{S_{xy}} - 1$ such that $E(\varepsilon_4) = 0$

The estimator $\bar{y}_{MM2,RSS}$ in terms of ε_0, ε_1 and ε_4 can be written as

$$\bar{y}_{MM2,RSS} = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1) - \frac{1}{mr}\frac{S_{xy}(1+\varepsilon_4)}{\bar{X}} + \bar{Y}W_{yx(i)}$$

Taking expectation of both sides and putting the value of $E(\varepsilon_0)$, $E(\varepsilon_1)$, $E(\varepsilon_4)$ and $E(\varepsilon_0, \varepsilon_1)$ we get

$$E(\bar{y}_{MM2,RSS}) = \bar{Y} \qquad (4.2)$$

Now, the variance of an unbiased estimator $\bar{y}_{MM2,RSS}$ is given by

$$V(\bar{y}_{MM2,RSS}) = E[\bar{y}_{MM2,RSS} - E(\bar{y}_{MM2,RSS})]^2$$

$$\approx E\left[\bar{Y}(1+\varepsilon_0+\varepsilon_1+\varepsilon_0\varepsilon_1) - \frac{1}{mr}\frac{S_{xy}(1+\varepsilon_4)}{\bar{X}} + \bar{Y}W_{yx(i)} - \bar{Y}\right]^2$$

Substituting the value of $W_{yx(i)}$ and the terms involving ε_0 and or ε_1 in a degree greater than two will be negligible, we get

$$V(\bar{y}_{MM2,RSS}) \approx \bar{Y}^2E\left[\varepsilon_0^2 + \varepsilon_1^2 + 2\varepsilon_0\varepsilon_1\right]$$

$$= \bar{Y}^2\left[\theta C_y^2 - W_{y[i]}^2 + \theta C_x^2 - W_{x(i)}^2 + 2\theta\rho_{yx}C_yC_x - 2W_{yx(i)}\right]$$

$$= \bar{Y}^2\left[\theta\{C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x\} - \{W_{y[i]} + W_{x(i)}\}^2\right]$$

$$\Rightarrow V(\bar{y}_{MM2,RSS}) = \theta\{S_y^2 + \bar{R}^2S_x^2 + 2\bar{R}S_{yx}\} - \bar{Y}^2\{W_{y[i]} + W_{x(i)}\}^2 \quad (4.3)$$

5. EFFICIENCY COMPARISON

On comparing (1.5) and (1.7) with (3.4) and (4.3) respectively, we obtain

1) $V(\bar{y}_{HR}) - V(\bar{y}_{MM1,RSS}) = A_1 \geq 0$, where $A_1 = \left[\bar{Y}W_{y[i]} - \bar{R}\bar{X}W_{x(i)}\right]^2$

$\Rightarrow V(\bar{y}_{MM1,RSS}) \leq V(\bar{y}_{HR})$

2) $V(\bar{y}_{pu}) - V(\bar{y}_{MM2,RSS}) = A_2 \geq 0$, where $A_2 = \bar{Y}^2\left[W_{y[i]} + W_{x(i)}\right]^2$

$\Rightarrow V(\bar{y}_{MM2,RSS}) \leq V(\bar{y}_{pu})$

It is easily seen that the Variance of the suggested estimators given in (3.4) and (4.3) are always smaller than the estimator given in (1.5) and (1.7) respectively, because A_1 and A_2 both are non-negative values. As a result, it is shown that the proposed estimators $\bar{y}_{MM1,RSS}$ and $\bar{y}_{MM2,RSS}$ for the population mean using RSS are more efficient than the usual estimators $\bar{y}_{HR}$ and $\bar{y}_{pu}$ respectively.

6. NUMERICAL ILLUSTRATION

To compare efficiencies of various estimators of our study, here, we take a population of size N=50 on page 1111 (Appendix) from the book entitled "Advanced Sampling Theory with Applications", Vol.2, by Sarjinder Singh, published from Kluwer Academic Publishers. The example considers the data of Agricultural loans outstanding of all operating banks in different states of USA in 1997, where *y* is real estate farm loans (study variable) in $000 and *x* is the non-real estate loans (auxiliary variable) in $000.

Table 1: Variance of Unbiased Ratio product type estimators using SRS

Variance	$\bar{y}_{HR}$	$\bar{y}_{pu}$
SRS	13975.163	121472.85

For the above population, the parameters are summarized as below:

Y=27771.73, X=43908.12, $\bar{Y}$=555.43, $\bar{X}$=878.16, S^2_X=1176526, S^2_Y=3420215, C^2_X=1.5256, C^2_Y=1.1086 and ρ=±0.8038

From the above population, we took 100 ranked set samples with size *m*=4 and number of cycles, *r*=3, so that *n(=mr)*=12. For these 100 ranked set samples chosen, we have computed variances of our proposed unbiased estimators $\bar{y}_{MM1,RSS}$ and $\bar{y}_{MM2,RSS}$ which are given in table 2. Table 1 shows variances of estimators $\bar{y}_{HR}$ and $\bar{y}_{pu}$ given by Hartley and Ross (1954) and Singh Sarjinder (2003).

On comparing table 2 with table 1, we conclude that the variance of the suggested estimators are always smaller than the usual estimators given by Hartley and Ross (1954) and Singh Sarjinder (2003). As a result, it is shown that all the suggested new unbiased estimators $\bar{y}_{MM1,RSS}$ and $\bar{y}_{MM2,RSS}$ for the population mean using RSS are more efficient than the usual estimators $\bar{y}_{HR}$ and $\bar{y}_{pu}$ under SRS.

Table 2: Variance's of Proposed Unbiased Ratio product type estimators using RSS

Estimators → Ranked Set ↓ Samples No.	$\bar{y}_{MM1, RSS}$	$y_{MM2, RSS}$	Estimators → Ranked Set ↓ Samples No.	$\bar{y}_{MM1, RSS}$	$\bar{y}_{MM2, RSS}$
1	12438.37	67504.79	51	13384.969	30989.307
2	13016.98	77508.04	52	7673.2999	94102.936
3	13197.66	81842.91	53	8229.8136	5306.7547
4	13926.77	65437.36	54	10462.698	62919.763
5	13621.4	58836.23	55	13121.59	48377.592
6	11758.08	37353.03	56	1788.9669	56359.464
7	12861.19	75986.64	57	13972.768	87667.84
8	10303.52	45531.78	58	13708.463	24565.243
9	13758.86	18836.11	59	12251.149	72429.287
10	13961.07	53564.03	60	13412.185	70708.367
11	11181.7	11329.55	61	13718.709	42879.12
12	13928.53	93419.85	62	13966.616	28043.095
13	10661.16	71510.00	63	8492.6452	19338.93
14	13964.48	70951.83	64	13064.728	61695.328
15	13441.37	82216.32	65	13916.954	43101.077

Table 2: Continue...

Estimators → Ranked Set ↓ Samples No.	$\bar{y}_{MM1, RSS}$	$y_{MM2, RSS}$	Estimators → Ranked Set ↓ Samples No.	$\bar{y}_{MM1, RSS}$	$y_{MM2, RSS}$
16	13849.04	17334.1	66	13423.987	51207.477
17	13682.36	94344.18	67	13969.995	100939.01
18	13630.33	55259.63	68	13719.219	10522.279
19	13904.4	75042.92	69	10317.694	81589.925
20	13953.72	78630.22	70	9999.0069	8234.1264
21	11338.47	41829.16	71	12771.944	72271.916
22	13817.58	90927.91	72	13920.642	56268.398
23	13917.00	59239.04	73	13885.759	62615.736
24	13210.14	56272.85	74	13965.613	52399.699
25	10554.963	37732.531	75	13619.829	84682.861
26	13845.68	63812.12	76	13869.975	66149.738
27	13580.94	29542.95	77	13167.006	29482.688
28	13029.55	55426.9	78	12780.679	73843.50
29	8896.086	71684.00	79	11786.12	79731.399
30	13952.28	30816.61	80	13904.675	62628.216
31	10606.55	81715.83	81	10683.328	59195.34
32	12665.86	65744.28	82	13122.578	76458.075
33	11799.00	68338.26	83	13649.028	19392.802
34	12326.32	85278.91	84	10493.371	12093.368
35	13879.64	85787.06	85	10594.885	58709.197
36	11371.68	42980.79	86	1724.0296	6737.3623
37	13791.64	56079.5	87	11877.095	39572.302
38	13382.85	85341.23	88	12803.454	47976.667
39	13463.82	45838.21	89	2030.7433	5452.3058
40	13814.98	34332.38	90	13963.529	86184.629
41	12955.451	64592.371	91	12690.16	64293.17
42	12901.017	101370.73	92	13678.99	51877.15
43	12597.964	30476.189	93	13973.97	47696.66
44	13897.197	91017.421	94	9765.359	18819.96
45	12565.832	34171.46	95	13888.91	45734.6
46	13861.178	43093.035	96	13966.06	75909.07
47	13810.066	39579.873	97	12061.56	20667.06
48	9065.6192	40173.071	98	9639.522	38622.52
49	13005.704	63977.224	99	4182.734	35059.87
50	13904.234	19832.434	100	8516.951	54618.03

7. CONCLUSION

We have proposed two modified Methods to construct unbiased ratio and product type estimators using RSS and obtained variance of the proposed unbiased ratio and product type estimators. The variance of proposed estimators have been compared with the variance of SRS estimators and found these proposed estimators have smaller variance than corresponding estimators. This theoretical result has been supported by the above example.

8. REFERENCES

Bouza, Carlos N., 2008. Ranked set sampling for the product estimator. Revista Investigation Operational 29(3), 201–206.

Cochran, W.G., 1940. Some properties of estimators based on sampling scheme with varying probabilities. Australian Journal of Statistics 17, 22–28.

Hartley, H.O., Ross, A., 1954. Unbiased ratio estimators. Nature 174, 270–271.

Jeelani, M.I., Bouza, C.N., 2015. New ratio method of estimation under ranked set sampling. Revista Investigación Operational 36, 151–155.

Kadilar, C., Unyazici, Y., Cingi, H., 2009. Ratio estimator for the population mean using ranked set sampling, Statistical Papers 50, 301–309.

McIntyre, G.A., 1952. A method of unbiased selective sampling using ranked sets, Australian Journal of Agricultural Research 3, 385–390.

Mandowara, V.L., Mehta, N., 2013. Efficient generalized ratio-product type estimators for finite population mean with ranked set sampling. Austrian Journal of Statistics 42(2), 137–148.

Mehta, N., Mandowara, V.L., 2012. A better estimator of population mean with power transformation based on ranked set sampling. Statistics in Transition-new Series 13(3), 551–558.

Mehta, N., Mandowara, V.L., 2016. A Modified ratio-cum-product estimator of finite population mean using ranked set sampling. Communication and Statistics-Theory and Methods 45(2), 267–276.

Mehta, N., Mandowara, V.L., 2020. Conflicts of interests - a general procedure for estimating finite population mean using ranked set sampling. Revista Investigacion Operacional 41(6), 902–903.

Murthy, M.N., 1964. Product method of estimation. Sankhya-A 26, 69-74.

Prasad, B., 1989. Some improved ratio type estimators of population mean and ratio in finite population sample surveys. Communication and Statistics-Theory and Methods 18, 379–392.

Quenouille, M.H., 1956. Notes on bias in estimation. Biometrika 43, 353–360.

Samawi, H.M., Muttlak, H.A., 1996. Estimation of ratio using rank set sampling. The Biometrical Journal 38, 753–764.

Singh, S., 2003. Advanced sampling theory with application. Vol. I, Kluwer Academic Publishers, Netherlands.