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Generalized Power Transformation Estimators in Stratified Ranked Set Sampling using Auxiliary Information

Nitu Mehta (Ranka) and Latika Sharma

Dept. of Agricultural Economics & Management, Rajasthan College of Agriculture, Maharana Pratap University of Agriculture and Technology, Udaipur, Rajasthan (313 001), India

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Corresponding nitumehta82@gmail.com

0000-0002-0883-3467

ABSTRACT

Auxiliary variable is commonly used in survey sampling to improve the precision of estimates. Whenever there is auxiliary information available, the researchers want to utilize it in the method of estimation to obtain the most efficient estimator. Stratified simple random sampling (SSRS) is used in certain types of surveys because it combines the conceptual simplicity of simple random sampling (SRS) with potentially significant gains in efficiency. It is a convenient technique to use whenever we wish to ensure that our sample is representative of the, population and also to obtain separate estimates for parameters of each sub domain of the population. Stratified Ranked Set Sampling combines the advantages of Stratification and Ranked set sampling (RSS) to obtain an unbiased estimator for the population mean, with potentially significant gains in efficiency. Under Stratified ranked set sampling scheme, we have suggested two general estimators using power transformation to estimate the population mean of the study variable. These methods are highly beneficial to the estimation based on Stratified Simple Random Sampling (SSRS). The first order approximation to the bias and mean square error (MSE) of the proposed estimators are obtained. Theoretically, it is shown that these suggested estimators are more efficient than the estimators in Stratified simple random sampling. A numerical illustration is also included to demonstrate the merits of the proposed estimator using SRSS over the corresponding estimators in SSRS.

KEYWORDS: Power transformation estimator, ratio estimator, RSS, SRSS

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1. INTRODUCTION

Auxiliary variable is commonly used in survey sampling to improve the precision of estimates. In some cases, in addition to mean of auxiliary variable, various parameters related to auxiliary variable such as standard deviation, coefficient of variation, skewness, kurtosis, correlation coefficient, etc. may also be known. For these cases, many authors such as Upadhyaya and Singh (1999), Sisodia and Dwivedi (1981), Singh and Tailor (2003) developed various estimators to improve the ratio estimators in the simple random sampling. Mehta and Mandowara (2020, 2022) modified Some Efficient Methods to Remove Bias in Ratio and Product Types Estimators in Ranked Set Sampling. Kadilar and Cingi (2003) adapted the estimators in Upadhyaya and Singh (1999) to the stratified random sampling. Singh et al. (2008) suggested two modified estimators of population mean using power transformation in Stratified random sampling. Ranked set sampling (RSS) was first suggested by McIntyre (1952) and its use in Stratified Sampling was introduced by Samawi (1996) to increase the efficiency of estimator of population mean. Mandowara and Mehta (2014a, 2014b) Suggested Modified ratio estimators using stratified ranked set sampling and Efficient ratio-cum-product estimator using stratified ranked set sampling. The performance of the combined and the separate ratio estimates using the stratified ranked set sample (SRSS) was given by Samawi and Siam (2003). Here we shall propose two generalized estimators of population mean using power transformation using SRSS based on auxiliary variable.

In Ranked set sampling (RSS), r independent random sets, each of size r and each unit in the set being selected with equal probability and without replacement, are selected from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the r^{th} set. This cycle may be repeated m times, so mr (=n) units have been measured during this process. From the h^{th} stratum of the population, first choose r_h independent samples each of size r_h $h = 1, 2, \ldots, L$. Rank each sample, and use RSS scheme to obtain L independent RSS samples of size r_h, one from each stratum. Let $r_1 + r_2 + \ldots\ldots + r_L = r$. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ *elements* have been obtained. A modification of the above procedure is suggested here to be used for the estimation of the ratio using stratified ranked set sample. For the h^{th} stratum, first choose r_h independent samples, each of size r_h of bivariate elements from the h^{th} subpopulation (stratum), $h = 1, 2, \ldots, L$. Rank each sample with respect to one of the variables, say Y or X. Then use the RSS sampling scheme to obtain L independent RSS samples of size r_h one from each stratum. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ bivariate elements have been obtained. We will use the following notation for the stratified ranked set sample when the ranking is on the variable X. For the k^{th} cycle and the h^{th} stratum, the SRSS is denoted by, $\{(Y_{h[1]k}, X_{h(1)k})(Y_{h[2]k}, X_{h(2)k}), \ldots\ldots\ldots (Y_{h[r_h]k}, X_{h(r_h)k}) : k = 1,2,..m; h = 1,2,..L\}$ where $Y_{h[i]k}$ is the i^{th} Judgment ordering in the i^{th} set for the study variable and $X_{h(i)k}$ is the i^{th} order statistic in the i^{th} set for the auxiliary variable. We should note that $E(Y_{h[i]k}) = \mu_{y_{h[i]k}}$ and $E(X_{h(i)k}) = \mu_{x_{h(i)k}}$.

2. ESTIMATORS IN STRATIFIED RANDOM SAMPLING AND STRATIFIED RANKED SET SAMPLING

The usual ratio estimator given by Cochran (1977) for the population mean $\bar{Y}$ in stratified random sampling is defined by

$$\bar{y}_{SSRS} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{1}$$

Where $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$ are the unbiased estimators of population mean $\bar{Y}$ and $\bar{X}$ respectively.

When the population coefficient of variation C_x is known, Motivated by Sisodia and Dwivedi (1981), Kadilar and Cingi (2003) suggested a modified ratio estimator for $\bar{Y}$ in stratified random sampling as

$$\bar{y}_{stSD} = \bar{y}_{st}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{x_h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{x_h}\right)} \tag{2}$$

Motivated by Kadilar and Cingi (2003) developed ratio-type estimator for $\bar{Y}$ as

$$\bar{y}_{stSK} = \bar{y}_{st}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)} \tag{3}$$

Kadilar and Cingi (2003) considered ratio type estimators based on Upadhyaya and Singh (1999), using both coefficient of variation and kurtosis in stratified random sampling as

$$\bar{y}_{stUS1} = \bar{y}_{st}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{x_h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{x_h}\right)} \tag{4}$$

$$\bar{y}_{stUS2}=\bar{y}_{st}\frac{\sum_{h=1}^{L}W_h\left(\bar{X}_hC_{x_h}+\beta_{2h}(x)\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_hC_{x_h}+\beta_{2h}(x)\right)} \quad (5)$$

By applying power transformation on Upadhyaya and Singh (1999) estimators, Singh (2008) suggested the modified estimators given as

$$\bar{y}_{stUS1(\alpha_1)}=\bar{y}_{st}\left(\frac{\sum_{h=1}^{L}W_h\left(\bar{X}_h\beta_{2h}(x)+C_{x_h}\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_h\beta_{2h}(x)+C_{x_h}\right)}\right)^{\alpha_1} \quad (6)$$

$$\bar{y}_{stUS2(\alpha_2)}=\bar{y}_{st}\left(\frac{\sum_{h=1}^{L}W_h\left(\bar{X}_hC_{x_h}+\beta_{2h}(x)\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_hC_{x_h}+\beta_{2h}(x)\right)}\right)^{\alpha_2} \quad (7)$$

Where α_1 and α_2 are suitably chosen scalars such that the mean squared errors of $\bar{y}_{stUS1(\alpha_1)}$ and $\bar{y}_{stUS2(\alpha_2)}$ are minimum.

To the first degree of approximation the mean squared error (MSE) of the estimators $\bar{y}_{SSRS}$, $\bar{y}_{stSD}$, $\bar{y}_{stSK}$, $\bar{y}_{stUS1}$, $\bar{y}_{stUS2}$, $\bar{y}_{stUS1(\alpha_1)}$ and $\bar{y}_{stUS2(\alpha_2)}$ respectively are

$$MSE(\bar{y}_{SSRS})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2S_{x_h}^2-2RS_{x_hy_h}\right) \quad (8)$$

$$MSE(\bar{y}_{stSD})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\lambda_1^2S_{x_h}^2-2R\lambda_1S_{x_hy_h}\right) \quad (9)$$

$$MSE(\bar{y}_{stSK})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\lambda_2^2S_{x_h}^2-2R\lambda_2S_{x_hy_h}\right) \quad (10)$$

$$MSE(\bar{y}_{stUS1})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\gamma_1^2S_{x_h}^2-2R\gamma_1S_{x_hy_h}\right) \quad (11)$$

$$MSE(\bar{y}_{stUS2})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\gamma_2^2S_{x_h}^2-2R\gamma_2S_{x_hy_h}\right) \quad (12)$$

$$MSE(\bar{y}_{stUS1(\alpha_1)})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\varphi_1^2\alpha_1^2S_{x_h}^2-2R\varphi_1\alpha_1S_{x_hy_h}\right) \quad (13)$$

$$MSE(\bar{y}_{stUS2(\alpha_2)})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(S_{y_h}^2+R^2\varphi_2^2\alpha_2^2S_{x_h}^2-2R\varphi_2\alpha_2S_{x_hy_h}\right) \quad (14)$$

where $\lambda_1=\frac{\sum_{h=1}^{L}W_h\bar{X}_h}{\sum_{h=1}^{L}W_h(\bar{X}_h+C_{x_h})}$, $\lambda_2=\frac{\sum_{h=1}^{L}W_h\bar{X}_h}{\sum_{h=1}^{L}W_h(\bar{X}_h+\beta_{2h}(x))}$, $\gamma_1=\varphi_1=\frac{\sum_{h=1}^{L}W_h\bar{X}_h\beta_{2h}(x)}{\sum_{h=1}^{L}W_h(\bar{X}_h\beta_{2h}(x)+C_{x_h})}$,

$\gamma_2=\varphi_2=\frac{\sum_{h=1}^{L}W_h\bar{X}_hC_{x_h}}{\sum_{h=1}^{L}W_h(\bar{X}_hC_{x_h}+\beta_{2h}(x))}$, $S_{y_h}^2=\frac{\sum_{i=1}^{N_h}(Y_{hi}-\bar{Y}_h)^2}{N_h-1}$, $S_{x_h}^2=\frac{\sum_{i=1}^{N_h}(X_{hi}-\bar{Y}_h)^2}{N_h-1}$ and

$S_{y_hx_h}=\frac{\sum_{i=1}^{N_h}(Y_{hi}-\bar{Y}_h)(X_{hi}-\bar{X}_h)}{N_h-1}$.

The combined ratio estimator of population mean $\bar{Y}$ given by Samawi and Siam (2003), using stratified ranked set sampling is defined as

$$\bar{y}_{SRSS}=\bar{y}_{[SRSS]}\left(\frac{\bar{X}}{\bar{x}_{(SRSS)}}\right) \quad (15)$$

where $\bar{y}_{[SRSS]}=\sum_{h=1}^{L}W_h\bar{y}_{h[r_h]}$ and $\bar{x}_{(SRSS)}=\sum_{h=1}^{L}W_h\bar{x}_{h(r_h)}$.

The Bias and MSE of the estimator $\bar{y}_{SRSS}$ to the first degree of approximation are respectively given by

$$B(\bar{y}_{SRSS})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2}-\frac{S_{x_hy_h}}{\bar{X}\bar{Y}}\right\}-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\sum_{i=1}^{r_h}D_{x_h(i)}^2-\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right\}\right] \quad (16)$$

$$MSE(\bar{y}_{SRSS})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2+R^2S_{x_h}^2-2RS_{x_hy_h}\right\}-\bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-D_{x_h(i)}\right)^2\right\}\right] \quad (17)$$

where $n_h=m_h$, $D_{y_h[i]}^2=\frac{\tau_{y_h[i]}^2}{\bar{Y}^2}$, $D_{x_h(i)}^2=\frac{\tau_{x_h(i)}^2}{\bar{X}^2}$ and $D_{x_h(i)y_h[i]}=\frac{\tau_{x_h(i)y_h[i]}}{\bar{Y}\bar{X}}$.

Here we would also like to remind that $\tau_{x_h(i)}=\mu_{x_h(i)}-\bar{X}_h$, $\tau_{y_h[i]}=\mu_{y_i[i]}-\bar{Y}_h$ and $\tau_{x_h(i)y_h[i]}=(\mu_{x_h(i)}-\bar{X}_h)(\mu_{y_h[i]}-\bar{Y}_h)$ where $\mu_{x_h(i)}=E[x_{h(i)}]$, $\mu_{y_h(i)}=E[y_{h(i)}]$, $\bar{X}_h$ and $\bar{Y}_h$ are the means of the h^{th} stratum for variables X and Y, respectively.

Motivated by Kadilar and Cingi (2003), Mandowara and Mehta (2014a) proposed ratio-type estimator for $\bar{Y}$ using stratified ranked set sampling, when the population coefficient of variation of auxiliary variable C_x is known

$$\bar{y}_{strMM1}=\bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L}W_h\left(\bar{X}_h+C_{x_h}\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_{h(r_h)}+C_{x_h}\right)} \quad (18)$$

$$\bar{y}_{strMM2}=\bar{y}_{[SRSS]}\frac{\sum_{h=1}^{L}W_h\left(\bar{X}_h+\beta_{2h}(x)\right)}{\sum_{h=1}^{L}W_h\left(\bar{x}_{h(r_h)}+\beta_{2h}(x)\right)} \quad (19)$$

where $\bar{y}_{[SRSS]}=\sum_{h=1}^{L}W_h\bar{y}_{h[r_h]}$ and $\bar{x}_{(SRSS)}=\sum_{h=1}^{L}W_h\bar{x}_{h(r_h)}$.

The bias and MSE's of the estimators $\bar{y}_{strMM1}$ and $\bar{y}_{strMM2}$ are as

$$B(\bar{y}_{strMM1})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{\lambda_1^2S_{x_h}^2}{\bar{X}^2}-\frac{\lambda_1S_{x_hy_h}}{\bar{X}\bar{Y}}\right\}-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda_1^2\sum_{i=1}^{r_h}D_{x_h(i)}^2-\lambda_1\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right\}\right] \quad (20)$$

$$MSE(\bar{y}_{strMM1})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2+R^2\lambda_1^2S_{x_h}^2-2R\lambda_1S_{x_hy_h}\right\}-\bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\lambda_1D_{x_h(i)}\right)^2\right\}\right] \quad (21)$$

and

$$B(\bar{y}_{strMM2})=\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{\lambda_2^2S_{x_h}^2}{\bar{X}^2}-\frac{\lambda_2S_{x_hy_h}}{\bar{X}\bar{Y}}\right\}-\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda_2^2\sum_{i=1}^{r_h}D_{x_h(i)}^2-\lambda_2\sum_{i=1}^{r_h}D_{x_h(i)y_h[i]}\right)\right\}\right] \quad (22)$$

$$MSE(\bar{y}_{strMM2})=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2+R^2\lambda_2^2S_{x_h}^2-2R\lambda_2S_{x_hy_h}\right\}-\bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]}-\lambda_2D_{x_h(i)}\right)^2\right\}\right] \quad (23)$$

Motivated by Kadilar and Cingi (2003), Mandowara and Mehta (2014b) suggested ratio –type estimators based on Upadhyaya and Singh (1999) in stratified ranked set sampling as

$$\bar{y}_{strMM3} = \bar{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2h}(x) + C_{x_h}\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(r_h)} \beta_{2h}(x) + C_{x_h}\right)} \quad (24)$$

$$\bar{y}_{strMM4} = \bar{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h(r_h) C_{x_h} + \beta_{2h}(x)\right)} \quad (25)$$

Again, bias and MSE's of the estimators $\bar{y}_{strMM3}$ and $\bar{y}_{strMM4}$ are as

$$\Rightarrow B(\bar{y}_{strMM3}) = \bar{Y}\left[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\left\{\frac{\gamma_1^2 S_{x_h}^2}{\bar{X}^2} - \frac{\gamma_1 S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\gamma_1^2 \sum_{i=1}^{r_h} D_{x_h(i)}^2 - \gamma_1 \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right)\right\}\right] \quad (26)$$

$$\Rightarrow MSE(\bar{y}_{strMM3}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2 \gamma_1^2 S_{x_h}^2 - 2R\gamma_1 S_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \gamma_1 D_{x_h(i)}\right)^2\right\}\right] \quad (27)$$

and

$$B(\bar{y}_{strMM4}) = \bar{Y}\left[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\left\{\frac{\gamma_2^2 S_{x_h}^2}{\bar{X}^2} - \frac{\gamma_2 S_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left\{\frac{m}{n_h}\left(\gamma_2^2 \sum_{i=1}^{r_h} D_{x_h(i)}^2 - \gamma_2 \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right)\right\}\right] \quad (28)$$

$$MSE(\bar{y}_{strMM4}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2 \gamma_2^2 S_{x_h}^2 - 2R\gamma_2 S_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \gamma_2 D_{x_h(i)}\right)^2\right\}\right] \quad (29)$$

4. MATERIAL AND METHODS

We propose generalized power transformation estimators are respectively given by

$$\bar{y}_{strMM1(\alpha_1)} = \bar{y}_{[SRSS]} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2h}(x) + C_{x_h}\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(r_h)} \beta_{2h}(x) + C_{x_h}\right)}\right)^{\alpha_1} \quad (30)$$

$$\bar{y}_{strMM2(\alpha_2)} = \bar{y}_{[SRSS]} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h(r_h) C_{x_h} + \beta_{2h}(x)\right)}\right)^{\alpha_2} \quad (31)$$

To obtain bias and MSE of $\bar{y}_{strMM1(\alpha_1)}$, we put $\bar{y}_{[SRSS]} = \bar{Y}(1+\delta_0)$ and $\bar{x}_{(SRSS)} = \bar{X}(1+\delta_1)$ so that $E(\delta_0) = E(\delta_1) = 0$.

Now $V(\delta_0) = E(\delta_0^2) = \frac{V(\bar{y}_{[SRSS]})}{\bar{Y}^2}$

$$= \sum_{h=1}^{L} W_h^2 \frac{1}{mr_h} \frac{1}{\bar{Y}^2}\left[S_{y_h}^2 - \frac{m}{n_h}\sum_{i=1}^{r_h} \tau_{y_h[i]}^2\right] = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_h[i]}^2\right]$$

Similarly, $$E(\delta_1^2) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right]$$

$$E(\delta_0, \delta_1) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right]$$

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\delta_0|$ and $|\delta_1|$ as small so that the terms involving δ_0 and or δ_1 in a degree greater than two will be negligible.

The bias of the estimator $\bar{y}_{strMM1(\alpha_1)}$ to the first degree of approximation, are respectively given by

$B(\bar{y}_{strMM1(\alpha_1)}) = E(\bar{y}_{strMM1(\alpha_1)}) - \bar{Y}$

Here $= \bar{y}_{strMM1(\alpha_1)} = \bar{Y}(1+\delta_0)(1+\varphi_1\delta_1)^{-\alpha_1}$

$$= \bar{Y}\left[(1+\delta_0)\left\{1 - \varphi_1\alpha_1\delta_1 + \frac{\alpha_1(\alpha_1+1)}{2}\varphi_1^2\delta_1^2 + o(\delta_1)\right\}\right]$$

$$B(\bar{y}_{strMM1(\alpha_1)}) = \bar{Y}\left[\varphi_1^2 \frac{\alpha_1(\alpha_1+1)}{2}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right) - \varphi_1\alpha_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right]\right] \quad (32)$$

The MSE of the estimator $\bar{y}_{strMM1(\alpha_1)}$ to the first degree of approximation, are respectively given by

$$MSE(\bar{y}_{strMM1(\alpha_1)}) = E(\bar{y}_{strMM1(\alpha_1)} - \bar{Y})^2$$

$$= \bar{Y}^2 E\left[\delta_0 - \varphi_1\alpha_1\delta_1 + \varphi_1^2 \frac{\alpha_1(\alpha_1+1)}{2}\delta_1^2 - \varphi_1\alpha_1\delta_0\delta_1\right]^2$$

$$= \bar{Y}^2 E\left[\delta_0^2 + \varphi_1^2\alpha_1^2\delta_1^2 - 2\varphi_1\alpha_1\delta_0\delta_1\right]$$

$$= \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{y_h}^2}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_h[i]}^2\right\} + \varphi_1^2\alpha_1^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right\} - 2\varphi_1\alpha_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right\}\right]$$

$$\Rightarrow MSE(\bar{y}_{strMM1(\alpha_1)}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\varphi_1^2\alpha_1^2 S_{x_h}^2 - 2R\varphi_1\alpha_1 S_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \varphi_1\alpha_1 D_{x_h(i)}\right)^2\right\}\right] \quad (33)$$

Similarly, bias and mean squared error of the estimator $\bar{y}_{strMM2(\alpha_2)}$ can be obtained respectively by changing the place of coefficient of kurtosis and coefficient of variation as

$$B(\bar{y}_{strMM2(\alpha_2)}) = \bar{Y}\left[\varphi_2^2 \frac{\alpha_2(\alpha_2+1)}{2}\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right) - \varphi_2\alpha_2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right]\right] \quad (34)$$

$$MSE(\bar{y}_{strMM2(\alpha_2)}) = \sum_{h=1}^{L}\frac{W_h^2}{n_h}\left[\left\{S_{y_h}^2 + R^2\varphi_2^2\alpha_2^2 S_{x_h}^2 - 2R\varphi_2\alpha_2 S_{x_h y_h}\right\} - \bar{Y}^2\left\{\frac{m}{n_h}\sum_{i=1}^{r_h}\left(D_{y_h[i]} - \varphi_2\alpha_2 D_{x_h(i)}\right)^2\right\}\right] \quad (35$$

The optimum values of α_1 and α_2 to minimize the MSE's of $\bar{y}_{strMM1(\alpha_1)}$ and $\bar{y}_{strMM2(\alpha_2)}$ respectively, can easily be found as follows

$$\frac{\partial MSE(\bar{y}_{strMM1(\alpha_1)})}{\partial \alpha_1} = 0$$

$$\Rightarrow \alpha_1^* = \frac{\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right\}}{\varphi_1\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right\}} \quad (36)$$

Similarly, $$\alpha_2^* = \frac{\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}\right\}}{\varphi_2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left\{\frac{S_{x_h}^2}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i)}^2\right\}} \quad (37)$$

To compare efficiencies of various estimators of our study, here, we take a Stratified population with 3 strata of sizes 12, 30 & 17 respectively of page 1119 (Appendix) from the book entitled "Advanced Sampling Theory with Applications", Vol.2, by Sarjinder Singh published from Kluwer Academic Publishers. The example considers the data of Tobacco for Area and Production in specified countries during 1998, where y is production (study variable) in metric tons and x is area (auxiliary variable) in hectares.

For the above population, the parameters are summarized as below:

For total population, N=59, $\overline{Y} = 76485.42$, $\overline{X} = 26942.29$

Table 1: Population Parameters

Stratum-1	Stratum-2	Stratum-3
$N_1 = 12$	$N_2 = 30$	$N_3 = 17$
$n_1 = 9$	$n_2 = 15$	$n_3 = 12$
$W_1 = 0.2034$	$W_2 = 0.5085$	$W_3 = 0.2881$
$\overline{X}_1 = 5987.83$	$\overline{X}_2 = 11682.73$	$\overline{X}_3 = 68662.29$
$\overline{Y}_1 = 11788$	$\overline{Y}_2 = 16862.27$	$\overline{Y}_3 = 227371.53$
$S^2_{x_1} = 278428105$	$S^2_{x_2} = 760238523$	$S^2_{x_3} = 12187889050$
$S^2_{y_1} = 153854583$	$S^2_{y_2} = 2049296094$	$S^2_{y_3} = 372428238550$
$S_{y_1x_1} = 628461731$	$S_{y_2x_2} = 1190767859$	$S_{y_3x_3} = 27342963562$
$C_{x_1} = 0.8812$	$C_{x_2} = 2.3601$	$C_{x_3} = 1.6079$
$\beta_{21}(x) = 1.8733$	$\beta_{22}(x) = 10.7527$	$\beta_{21}(x) = 8.935$
$R_1 = 1.97$	$R_2 = 1.44$	$R_3 = 3.31$

5. RESULTS AND DISCUSSION

We took ranked set samples of sizes $r_1 = 3$, $r_2 = 5$ & $r_3 = 4$ from stratum 1st, 2nd and 3rd respectively. Further each ranked set sample from each stratum was repeated with number of cycles $m = 3$. So that sample sizes of stratified ranked set samples are equivalent to stratified simple random samples with n_h $(=mr_h)$ for the h^{th} stratum, $h = 1, 2, 3$.

On comparing (13) and (14) with (33) and (35) respectively, we obtained

1) $MSE(\overline{y}_{stUS1(\alpha_1)}) - MSE(\overline{y}_{strMM1(\alpha_1)}) = A_1 \geq 0$, where $|A_1 = \overline{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{n_h} \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \varphi_1 \alpha_1 D_{x_h(i)}\right)^2$

$$\Rightarrow MSE(\overline{y}_{strMM1(\alpha_1)}) \leq MSE(\overline{y}_{stUS1(\alpha_1)})$$

2) $MSE(\overline{y}_{stUS2(\alpha_2)}) - MSE(\overline{y}_{strMM2(\alpha_2)}) = A_2 \geq 0$, where $A_2 = \overline{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{n_h} \frac{m}{n_h} \sum_{i=1}^{r_h} \left(D_{y_h[i]} - \varphi_2 \alpha_2 D_{x_h(i)}\right)^2$

$$\Rightarrow MSE(\overline{y}_{strMM2(\alpha_2)}) \leq MSE(\overline{y}_{stUS2(\alpha_2)})$$

It is easily seen that the MSE of the suggested estimators given in (30) and (31) are always smaller than the estimator given in (6) and (7) respectively, because A_1 and A_2 all are non-negative values. As a result, show that the power transformed estimators for the population mean in Stratified ranked set sampling are more efficient than the corresponding usual estimators of stratified sampling.

The estimated relative efficiencies of various proposed Stratified ranked set estimators in comparison with corresponding Stratified SRS estimators are as shown in the next table:

In the table 2, we see that the proposed Stratified ranked set estimators with power transformation are more efficient than corresponding Stratified SRS estimators.

Table 2: Comparison of various estimators

Variances of various stratified SRS estimators	$\overline{y}_{stUS1(\alpha_1)} = \overline{y}_{stUS2(\alpha_2)}$ 2158910787
Variances of corresponding Stratified ranked set sampling es-timators	$\overline{y}_{strMM3(\alpha_1)} = \overline{y}_{strMM4(\alpha_2)}$ 1605469167
Relative Effi-ciencies in %	134.4723

6. CONCLUSION

We proposed two general estimators using power transformation to estimate the population mean and obtained its MSE equation. The MSE of proposed estimator was compared with corresponding stratified simple random sampling estimator and found that the proposed estimator had smaller MSE than the corresponding estimator. This theoretical result was supported by the above example. With this conclusion, we hope to develop new estimators in other sampling methods in the forthcoming studies.

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